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An Exact Penalty Function Approach for Solving the Linear Bilevel Multiobjective Programming Problem

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Abstract. In this paper, a new penalty function approach is proposed for the linear bilevel multiobjective programming problem. Using the optimality conditions of the lower level problem, we transform the linear bilevel multiobjective programming problem into the corresponding linear multiobjective programming problem with complementary constraint. The complementary constraint is appended to the upper level objectives with a penalty. Then, we give via an exact penalty method an existence theorem of Pareto optimal solutions and propose an algorithm for the linear bilevel multiobjective programming problem. Numerical results showing viability of the penalty function approach are presented.

1. Introduction

Bilevel programming(BP), which is characterized by the existence of two optimization problems in which the constraint region of the first-level problem is implicitly determined by another optimization problem, has increasingly been addressed in literature, both from the theoretical and computational points of view(see the monographs of Dempe[1] and Bard[2] and the bibliography reviews by Vicente[3], Dempe[4] and Colson[5]). In the last two decades, many papers have been published about bilevel optimization, however there are only very few of them dealing with bilevel multiobjective programming(BMP) problem, where the upper level or the lower level or both of a bilevel decision have multiple conflicting objectives[6–8].

Shi and Xia[9, 10] propose an interactive algorithm based on the concepts of satisfactoriness and direction vector for nonlinear bilevel multiobjective problem. Abo-Sinna[11], Osman et al.[12] present some approaches via fuzzy set theory for solving bilevel and multiple level multiobjective problem, and Teng[13], Deb and Sinha[14] give evolutionary algorithms for some bilevel multiobjective programming problems. Besides, Bonnel and Morgan[15], Zheng and Wan[16] consider a so-called semivector bilevel optimization problem and propose solution methods based on penalty approach. A recent study by Eichfelder[8] suggests a refinement based strategy in which the algorithm starts with a uniformly distributed set of points on upper level variable. Noted that if the dimension of upper level variable is high, generating a uniformly spread upper level variables and refining the resulting upper level variable will be computationally expensive.

The linear bilevel multiobjective programming(LBMP) problem, i.e., both the objective functions and the constraint functions are linear functions, has attracted more and more attention. Nishizaki and Sakawa[17]

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give three Stackelberg solution definitions and propose the corresponding algorithms based on the idea of the K-th best method; Ankhili and Mansouri[18] propose an exact penalty function algorithm based on the marginal function of lower level problem for the LBMP problem, where the upper level is a linear scalar optimization problem and the lower level is a linear multiobjective optimization problem; Calvete and Gale[19] analyze the characters of the feasible region and give some algorithms frame for the LBMP problem, where the upper level is linear multiobjective optimization problem and the lower level is linear scalar optimization problem.

In this paper, different from the solving approaches mentioned above, we propose a new exact penalty algorithm for the linear bilevel multiobjective programs, where both the upper level and the lower level are linear multiobjective optimization problem. Our strategy can be outlined as follows. By using the weighted sum scalarization approach and the KKT conditions, we reformulate the LBMP problem as a linear multiobjective optimization problem with complementary constraint. Thereafter, we append the complementary constraint to the upper level objectives with a penalty, and construct a penalized problem for the LBMP problem. Then we give via an exact penalty method an existence theorem of Pareto optimal solutions and propose an algorithm for the LBMP problem. Finally, we give some numerical examples to illustrate the algorithm proposed in this paper.

The remainder of the paper is organized as follows. In the next section we give the mathematical model of the LBMP problem and construct the penalized problem. In Section 3, we analyze the characters of the penalized problem and give via an exact penalty method an existence theorem of Pareto optimal solutions. In Section 4, we propose the algorithm and give the numerical results. Finally, we conclude the paper with some remarks.

2. Linear Bilevel Multiobjective Programming and Penalized Problem

The linear bilevel multiobjective programming problem, which is considered in this paper, can be written as:

$$\begin{aligned} &\max_{x\geq 0} Cx + C'y \\ &s.t.\ \max_{y\geq 0} Dy \\ &\qquad s.t.\ A_1x + A_2y \leq b, \end{aligned} \tag{1}$$

where $x \in R^n, y \in R^m, b \in R^p, A_1 \in R^{p\times n}, A_2 \in R^{p\times m}, C \in R^{q\times n}, C' \in R^{q\times m}, D \in R^{l\times m}$.

Let $S = \{(x, y)|A_1x + A_2y \leq b, x \geq 0, y \geq 0\}$ denote the constraint region of problem (1), $\bar{S} = \{y \in R^m_+|A_1x+A_2y \leq b\}$ denote the feasible set of the lower level problem, and $\Pi_x = \{x \in R^n_+|\exists y \in R^m_+, A_1x+A_2y \leq b\}$ be the projection of S onto the decision space of the upper level problem.

To well define problem (1), we make the following assumption:

(H_1) The constraint region S is nonempty and compact.

(H_2) For any $(x, y) \in S$, the vectors $A_{2i}, i \in I = \{i|A_{1i}x + A_{2i}y = b_i,\ i = 1, 2, \ldots, p, p \leq m\}$ are linear independence.

Remark 2.1. It is noted that the assumption (H_2) plays a key role in the method of replacing the lower level problem with its KKT optimality conditions[20]. Based on assumption (H_2), we can adopt the method of replacing the lower level problem with its KKT optimality conditions, and transform the bilevel programming into the corresponding single level programming.

For fixed $x \in R^n_+$, let $S(x)$ denote the weak efficiency set of solutions to the lower level problem:

$$\begin{aligned} (P_x):\ &\max_{y\geq 0} Dy \\ &s.t.\ A_2y \leq b - A_1x. \end{aligned}$$

Definition 2.1 A point (x, y) is feasible for problem (1) if $(x, y) \in S$ and $y \in S(x)$; the term (x^*, y^*) is a Pareto optimal solution to problem (1), provided that it is a feasible point and there exists no other feasible point (x, y) such that $Cx^* + C'y^* \leqq Cx + C'y$ and $Cx^* + C'y^* \neq Cx + C'y$.

Noted that for fixed $x \in \Pi_x$, the lower level problem (P_x) is the linear multiobjective programs. Then, for fixed $x \in \Pi_x$ we can get some Pareto optimal solution to the lower level problem (P_x) by solving the following scalarization problem

$$\begin{aligned}&\max_{y\geq 0} \lambda^T Dy\\ &s.t.\;\; A_2 y \leq b - A_1 x,\end{aligned}$$

where λ is some constant vector, i.e, the weight of the lower level objectives, which reflects the preference of the decision maker to the lower level objectives and belongs to the set $\Omega = \{\lambda | \lambda \in R^l_+, \sum_{i=1}^{l} \lambda_i = 1\}$.

Then, some Pareto optimal solutions of problem (1), which corresponds to some fixed λ, the weight of the lower level objectives, can be obtained by solving the following bilevel multiobjective programs, where the lower level problem is a scalar optimization problem

$$\begin{aligned}&\max_{x\geq 0} Cx + C'y\\ &s.t.\;\; \max_{y\geq 0} \lambda^T Dy\\ &\qquad s.t.\;\; A_1 x + A_2 y \leq b.\end{aligned} \tag{2}$$

Following assumption (H_2), we replace the lower level problem with its KKT optimality conditions and get the following programs

$$\begin{aligned}&\max Cx + C'y\\ &s.t.\;\; A_1 x + A_2 y + w = b,\\ &\qquad A_2^T u - v = D^T \lambda,\\ &\qquad u^T w + v^T y = 0,\\ &\qquad x, y, u, v, w \geq 0,\end{aligned} \tag{3}$$

where $u \in R^p, v \in R^m$ are the lagrangian multiplies, and $w \in R^p$ is slack variable.

For each $x \in \Pi_x$, let $y \in \gamma(x) = \arg\max_{y}\{\lambda^T Dy | A_1 x + A_2 y \leq b, y \geq 0\}$. We have the following proposition.

Proposition 2.1 Let $(\bar{x}, \bar{y})$ be a Pareto optimal solution of problem (2), and $(\bar{u}, \bar{v})$, $\bar{w}$ are corresponding multiplier and slack variable associated with $(\bar{x}, \bar{y})$ respectively. Then $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ is a Pareto optimal solution of problem (3). Conversely, suppose that $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ is a Pareto optimal solution to problem (3). Then $(\bar{x}, \bar{y})$ is a Pareto optimal solution of problem (2).

Proof. If $(\bar{x}, \bar{y})$ be a Pareto optimal solution of problem (2), it means that $\bar{y} \in \gamma(\bar{x})$. Then, there exists corresponding multiplier $(\bar{u}, \bar{v})$ and slack variable $\bar{w}$ such that the KKT optimality conditions of the lower level problem in problem (2) are satisfied in $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$, that's, $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ is feasible to problem (3). Now, suppose that $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ is not a Pareto optimal solution to problem (3), then there exists some feasible point $(\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}, \tilde{w})$, such that

$$C\bar{x} + C'\bar{y} \leqq C\tilde{x} + C\tilde{y} \text{ and } C\bar{x} + C'\bar{y} \neq C\tilde{x} + C\tilde{y}$$

It contradicts with $(\bar{x}, \bar{y})$ is a Pareto optimal solution to problem (2).

Now, we prove the second part of Proposition 2.1.

If $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ is a Pareto optimal solution to problem (3), then there doesn't exist other feasible point $(\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}, \tilde{w})$, such that

$$C\bar{x} + C'\bar{y} \leqq C\tilde{x} + C\tilde{y} \text{ and } C\bar{x} + C'\bar{y} \neq C\tilde{x} + C\tilde{y}$$

As $(\bar{x}, \bar{y})$ and $(\tilde{x}, \tilde{y})$ are both feasible to problem (2), following the above formula, $(\bar{x}, \bar{y})$ is a Pareto optimal solution to problem (2). The proof is completed. □

For problem (3), we append the complementary constraint to the upper level objectives with a penalty, then the following penalized problem is obtained

$$\begin{aligned}&\max F(x,y,u,v,w,K) = Cx + C'y - K(u^T w + v^T y)e\\ &s.t.\ \ A_1 x + A_2 y + w = b,\\ &\qquad A_2^T u - v = D^T \lambda,\\ &\qquad x,y,u,v,w \geq 0,\end{aligned} \tag{4}$$

where $e \in R^q$ has all its components equal to unity, and K is a large positive constant. Clearly, problem (4) will reach optimality when $u^T w + v^T y \to 0$.

3. Theoretical Results

We will now analyze the main theoretical result, i.e., the exactness of the penalty function approach, which means we can get the Pareto optimal solutions of problem (2) by solving the penalized problem (4) for some finite positive constant K.

Before presenting some theoretical results, we introduce some useful notations firstly. Let $Z = \{(x,y,w)|A_1x + A_2y + w = b, x \geq 0, y \geq 0, w \geq 0\}$, $W = \{(u,v)|A_2^T u - v = D^T\lambda, u \geq 0, v \geq 0\}$, and we denote the extreme points of W and Z by W_v and Z_v, respectively.

Theorem 3.1 For a given value of $(u,v) \in W$ and fixed K, a Pareto optimal solution to the following programs

$$\begin{aligned}&\max_{(x,y,w)} F(x,y,u,v,w,K)\\ &s.t.\ \ (x,y,w) \in Z\end{aligned} \tag{5}$$

is achievable at some $(x^*,y^*,w^*) \in Z_v$.

Proof. Noted that for a fixed value of $(u,v) \in W$ and K, problem (5) is the linear multiobjective programs, then Theorem 3.1 is obvious. □

Theorem 3.1 yields the following theorem.

Theorem 3.2 For fixed K, a Pareto optimal solution to problem (4) is achievable in $Z_v \times W_v$ and $Z_v \times W_v = (Z \times W)_v$.

Proof. Let $(x^*,y^*,w^*) \in Z_v$ be a Pareto optimal solution to problem (4). As $F(x^*,y^*,u,v,w^*,K)$ is affine functions of (u,v), and W is a polytope, then the following problem

$$\begin{aligned}&\max_{(u,v)} F(x^*,y^*,u,v,w^*,K)\\ &s.t.\ \ (u,v) \in W\end{aligned}$$

will have Pareto optimal solutions $(u^*,v^*) \in W_v$. This proves the first part and the second part is obvious following Theorem 2 in [22]. □

The above theorem is based on a fixed value of K. We now show that a finite value of K would yield an exact Pareto optimal solution to the overall problem (4), where the penalty term $u^T w + v^T y$ becomes zero.

Theorem 3.3 There exists a finite value of K, K^* say, for which the Pareto optimal solution (x,y,u,v,w) to the penalty function problem (4) satisfies $u^T w + v^T y = 0$.

Proof. Suppose that (x^*,y^*,w^*) is the Pareto optimal solution to problem (2), the linear bilevel multiobjective problem, then the optimality conditions of lower level problem are satisfied. That means $(u^*)^T w^* + (v^*)^T y^* = 0$.

Let (x,y,w) be a Pareto optimal solution to problem (4), then there exists an index i, such that

$$C_i x + C_i' y - K(u^T w + v^T y) \geq C_i x^* + C_i' y^* - K((u^*)^T w^* + (v^*)^T y^*) = C_i x^* + C_i' y^*,$$

where C_i and C_i' are the i-th rows of C and C', respectively.

Thus,

$$0 \le u^T w + v^T y \le \frac{\max[C_i x + C_i' y - C_i x^* - C_i' y^*]}{K} \le \frac{k}{K'},$$

where k is some constant. Thus, as $K \to \infty$, $u^T w + v^T y \to 0$. However, Since $Z_v \times W_v$ is finite, $u^T w + v^T y = 0$ for some large finite value of K, say K^*. □

We now show that, by increasing K monotonically, we can achieve some Pareto optimal solutions of the linear bilevel multiobjective programming problem (1).

Theorem 3.4 The penalty function approach yields some Pareto optimal solutions to problem (1).

Proof. For problem (1), optimality is reached when one can get the optimal value of the vector $Cx + C'y$ and also satisfy the optimality conditions of the lower level problem which is achieved when $u^T w + v^T y = 0$. The later is achieved following the essence of penalty function methods for multiobjective programs[23] and at a finite K(by Theorem 3.3). □

4. Algorithm and Numerical Results

Now, based on the above theorems we can propose an exact penalty function algorithm for solving linear bilevel multiobjective programming problem (1).

Algorithm

Step 0. Choose an initial point $(x^0, y^0, u^0, v^0, w^0) \in W_v \times Z_v$, the weight of the lower level objectives, a positive constant $K > 1$ and stopping tolerance $\epsilon > 0$, and set $k := 1$.

Step 1. Find a solution $(x^k, y^k, u^k, v^k, w^k)$ of problem (4).

Step 2. If $(u^k)^T w^k + (v^k)^T y^k \le \epsilon$, stop; else goto Step 3.

Step 3. Set $(x^{k+1}, y^{k+1}, u^{k+1}, v^{k+1}, w^{k+1}) := (x^k, y^k, u^k, v^k, w^k)$, $k := k + 1$, $K := 5K$, and go to Step 1.

In Step 1, to facilitate the resolution of problem (4), we adopt the so-called "ideal points"approach[21] to get some Pareto optimal solution. The reason why we don't adopt other approaches to get the Pareto optimal front of problem (4) is that our primary aim in this paper is to explore the exact penalty method of transforming the linear bilevel multiobjective programs to some smooth multiobjective programs. Of course, the Pareto optimal front of the smooth multiobjective programs can be obtained using some appropriate approach.

In Step 2, the stopping criterion is standard, and is usually a part of any practical stopping criterion in commercial codes for the solution of smooth optimization problems. Based on the stopping criterion, we can find an approximate stationary point of problem (4).

The following theorem states the convergence of the above algorithm.

Theorem 4.1 Let assumption (H_1) and (H_2) be satisfied, then the last point in the sequence $\{(x^k, y^k)\}$, which is generated by the above algorithm, is a Pareto optimal solution to problem (1).

Proof. Following Theorem 3.3, we know that the penalty function is exact. Then, It means that the sequence $\{(x^k, y^k)\}$, which is generated by the above algorithm, is finite. Let (x^*, y^*) be the last point in the sequence $\{(x^k, y^k)\}$. Following Theorem 3.4, It obvious that (x^*, y^*) is a Pareto optimal solution to problem (1). □

To illustrate the above algorithm proposed, we consider the following linear bilevel multiobjective programming problems.

Example 1[6]

$$\begin{aligned}
&\min_{x \ge 0} F(x, y) = (-x + 2y, 2x - 4y)\\
&s.t. \ -x + 3y \le 4\\
&\quad \min_{y \ge 0} f(x, y) = (-x + 2y, 2x - y)\\
&\quad s.t. \ x - y \le 0\\
&\qquad -x - y \le 0
\end{aligned}$$

Table 1: Pareto optimal solution corresponding to different K

Examples in this paper	The fixed weight of the lower level objectives $\lambda = (\lambda_1, \lambda_2) = (0.5, 0.5)$		
	$K = 1000$	$K = 5000$	$K = 25000$
Exam.1	$(0, 0)$	$(0, 0)$	$(0, 0)$
Exam.2	$(144.2, 26.8, 2.97, 67.7, 0)$	$(144.2, 26.8, 2.97, 67.7, 0)$	$(144.2, 26.8, 2.97, 67.7, 0)$
Exam.3	$(0.6, 2.4, 0, 0)$	$(0.6, 2.4, 0, 0)$	$(0.6, 2.4, 0, 0)$

Table 2: Results in this paper comparing with that in the references

Examples in this paper	A Pareto optimal solution and the upper level objective value obtained in this paper	The Pareto optimal solution and the upper level objective value given in the references
Exam.1	$(x^*, y^*) = (0, 0)$ $F(x^*, y^*) = (0, 0)$	$(x^*, y^*) = (0, 0.5)$ $F(x^*, y^*) = (1, -2)$
Exam.2	$(x^*, y^*) = (144.2, 26.8, 2.97, 67.7, 0)$ $F(x^*, y^*) = (482.7, 1831.4)$	$(x^*, y^*) = (146.30, 28.94, 0, 67.93, 0)$ $F(x^*, y^*) = (474.7, 1850.1)$
Exam.3	$(x^*, y^*) = (0.6, 2.4, 0, 0)$ $F(x^*, y^*) = (5.4, 4.2)$	$(x^*, y^*) = (1.5, 1.5, 4.1, 3.4)$ $F(x^*, y^*) = (4.5, 6.0)$

Example 2[6]

$$\begin{aligned}
&\max_{x\geq 0} F(x,y) = (x_1 + 9x_2 + 10y_1 + y_2 + 3y_3, 9x_1 + 2x_2 + 2y_1 + 7y_2 + 4y_3)\\
&s.t.\ 3x_1 + 9x_2 + 9y_1 + 5y_2 + 3y_3 \leq 1039\\
&\quad -4x_1 - x_2 + 3y_1 - 3y_2 + 2y_3 \leq 94\\
&\quad \max_{y\geq 0} f(x,y) = (4x_1 + 6x_2 + 7y_1 + 4y_2 + 8y_3, 6x_1 + 4x_2 + 8y_1 + 7y_2 + 4y_3)\\
&\quad s.t.\ 3x_1 - 9x_2 - 9y_1 - 4y_2 \leq 61\\
&\qquad 5x_1 + 9x_2 + 10y_1 - y_2 - 2y_3 \leq 924\\
&\qquad 3x_1 - 3x_2 + y_2 + 5y_3 \leq 420
\end{aligned}$$

Example 3[24]

$$\begin{aligned}
&\max_{x\geq 0} F(x,y) = (x_1 + 2x_2, 3x_1 + x_2)\\
&s.t.\ x_1 + x_2 \leq 3\\
&\quad \max_{y\geq 0} f(x,y) = (y_1 + 3y_2, 2y_1 + y_2)\\
&\quad s.t.\ -x_1 + y_1 + y_2 \leq 6\\
&\qquad -x_2 + y_1 \leq 3\\
&\qquad x_1 + x_2 + y_2 \leq 8
\end{aligned}$$

In the above examples, we choose the fixed weight of the lower level objectives as $\lambda = (\lambda_1, \lambda_2) = (0.5, 0.5)$, and obtain the Pareto optimal solutions, which are presented in Table 1.

In table 2, we compare the upper level objective value obtained in this paper with that in the corresponding references. Following the vector partial order, it is obvious that the Pareto optimal solutions obtained in this paper does be the Pareto optimal solutions to the above three examples. Then, the exact penalty function approach proposed in this paper to the linear bilevel multiobjective programming problem is usefulness and viability.

5. Conclusion

In this paper, we introduce a new exact penalty function method for solving linear bilevel multiobjective programs. The method is based on replacing the lower level problem with its optimality conditions and appending the complementary constraint to the upper level objectives with a penalty. The numerical results reported illustrated that the exact penalty function method introduced in this paper can be numerically efficient.

It is noted that, besides its theoretical properties, the new algorithm proposed in this paper has one distinct advantage: it only requires the use of practicable algorithms for the solution of smooth multiobjective optimization problems, no other complex operations are necessary.

References

[1] S.Dempe, Foundations of Bilevel Programming, Nonconvex Optimization and Its Applications, Kluwer, Dordrecht, 2002.
[2] J.Bard, Practical Bilevel Optimization: Algorithm and Applications, Nonconvex Optimization and Its Applications, Kluwer, Dordrecht, 1998.
[3] L.Vicente, P.Calamai, Bilevel and multilevel programming: A bibligraphy review, Journal of Global Optimization 5(3)(1994):291–306.
[4] S.Dempe, Annotated bibliography on bilevel programming and mathematical programs with equilibrium constraints, Optimization 52(3)(2003)333–359.
[5] B.Colson, P.Marcotte, G.Savard, An overview of bilevel optimization, Annals of Operations Research 153(2007)235–256.
[6] G.Zhang, J.Lu, T.Dillon, Decentralized multi-objective bilevel decision making with fuzzy demands, Knowledge-Based Systems 20(5)(2007)495-507.
[7] J.Ye, Necessary optimality conditions for multiobjective bilevel programs, Mathematics of Operations Reseach 36(1)(2011)165-184.
[8] G.Eichfelder, Multiobjective bilevel optimization, Mathematical Programming 123(2010)419-449.
[9] X.Shi, H.Xia, Interactive bilevel multiobjective decision making, Journal of Operations Research Society 48(9)(1997)943–949.
[10] X.Shi, H.Xia, Model and interative algorithm of bilevel multiobjective decision making with multiple interconnected decision makers, Journal of Multi-Criteria Decision Analysis 10(2001)27-34.
[11] M.Abo-Sinna, A bilevel nonlinear multiobjective decision making under fuzziness, Journal of Operational Research Society of India 38(5)(2001)484-495.
[12] M.Osman, M.Abo-Sinna, et al, A multilevel nonlinear multiobjective decision making under fuzziness, Applied Mathematics and Computation 153(1)(2004)239-252.
[13] C.Teng, L.Li, et al, A class of genetic algorithms on bilevel multiobjective decision making problem, Journal of Systems Science and Systems Engineering 9(3)(2000)290–296.
[14] K.Deb, A.Sinha, Solving bilevel multi-objective optimization problems using evolutionary algorithms, Lecture Notes in Computer Science: Evolutionary Multi-criterion Optimization 5467(2009):110–124.
[15] H.Bonnel, J.Morgan, Semivectorial bilevel optimization problem: Penalty approach, Journal of Optimization Theory and Applications 131(3)(2006)365–382.
[16] Y.Zheng, Z.Wan. A solutiion method for semivectorial bilevel programming problem via penalty method, Journal of Applied Mathematics and Computing 37(2011)207–219.
[17] I.Nishizaki, M.Sakawa, Stackelberg solutions to multiobjective two-level linear programming problem, Journal of Optimization Theory and Applications 103(1)(1999)161–182.
[18] Z.Ankhili, A.Mansouri, An exact penalty on bilevel programs with linear vector optimization lower level, European Journal of Operational Research 197(2009)36–41.
[19] H.I.Calvete, C.Gale, Linear bilevel programs with multiple objectives at the upper level, Journal of Computational and Applied Mathematics 234(2010)950–959.
[20] S.Dempe, J.Dutta, Is bilevel programming a special case of a mathematical program with complementarity constraints, Mathematical Programming, Ser.A 131(2012)37–48.
[21] H.W.Tang, X.Z.Qin, Pratical Methods of Optimization, Dalian University of Technology Press, Dalian, China, 2004.
[22] G.Anandalingam, D.J.White, A solution for the linear static Stackelberg problem using penalty function, IEEE Transactions Automatic Control 35(1990)1170–1173.
[23] D.J.White, Multiobjective programming and penalty function, Journal of Optimization Theory and Applications 43(4)(1984)583–600.
[24] C.O.Pieume, P.Marcotte, L.P.Fotso, Solving bilevel linear multiobjective programming problems, American Journal of Operations Research 1(2011)214–219.